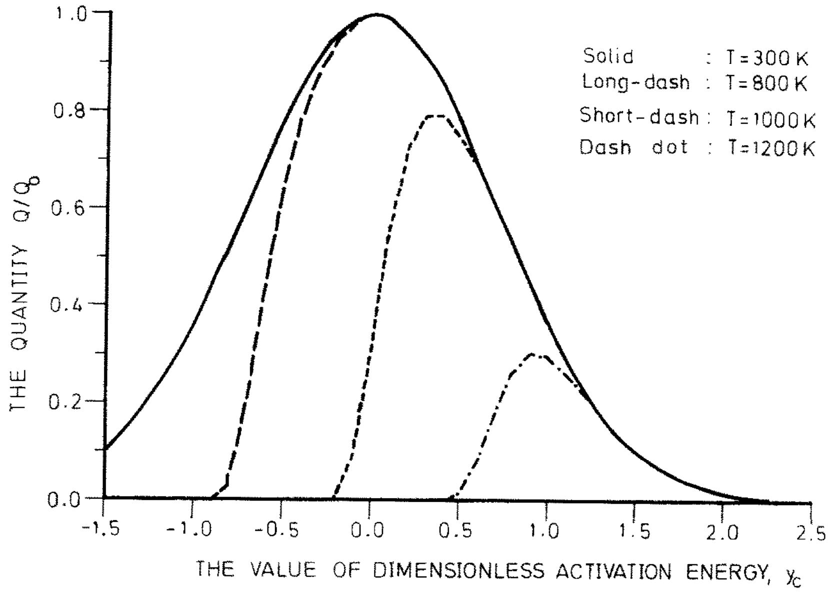

FIG. 2. Q/Q_0 curve.

vation energy, E_c, which is given by

$$E_c = X_c RT \quad \text{where} \quad X_c = \left(1-\frac{\ln\ln A}{1+\ln A}\right)\ln A. \tag{43}$$

Thus for the critical value of E_c one obtains

$$E_c = RT\left(1-\frac{\ln\ln A}{1+\ln A}\right)\ln A. \tag{44}$$

With this value of E_c in hand, one obtains the pyrolyzed material ratio, Q/Q_0, from equations (37) and (38). The graphical representation of Q/Q_0 for different temperatures can be seen in Fig. 2.

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Boundary effects on natural convection heat transfer for cylinders and cubes

R. O. WARRINGTON,† S. SMITH,‡ R. E. POWE§ and R. L. MUSSULMAN||

† Mechanical Engineering Department, Louisiana Tech University, Ruston, LA 71272, U.S.A.
‡ Mechanical Engineering Department, Montana State University, Bozeman, MT 59717, U.S.A.
§ Office of Research, Mississippi State University, Mississippi State, MS 39762, U.S.A.
|| Mechanical Engineering Department, California Polytechnic State University, San Luis Obispo, CA 93407, U.S.A.

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INTRODUCTION

MANY STUDIES have been done in the past concerning natural convection heat transfer into a fluid medium of infinite extent. More recently several studies have examined natural convection heat transfer to an enclosed fluid, where the convective motion is limited. Useful correlation equations have been developed for each case.

One difficulty often encountered in using these empirical equations is determining the range of gap width over which the equations for convection within an enclosure are applicable and when the equations for heat transfer into an infinite atmosphere apply. The object of this study was to examine heat transfer within an enclosure, increasing the gap width ratio over that studied previously to determine the bounds within which enclosure equations are applicable. The existing correlation equations were first analyzed to determine the range of gap width ratios which would most likely form the bounds for the two sets of equations. Bodies of varying sizes were then built to cover the range of gap width ratios required by the analysis.

The analysis showed that for Ra_b ranging from 10^5 to 10^{10}, the transition from the enclosure equations to the infinite atmosphere equations would occur within a range of gap width ratios between 2.5 and 5.0. The inner bodies built for use in the 27 cm cubical outer body included cubes ($2.28 \leqslant L/R_i \leqslant 4.25$) and cylinders ($2.84 \leqslant L/R_i \leqslant 5.89$). Four fluids were used; air, water, 20 cs silicone oil, and 96% aqueous glycerin, yielding Prandtl numbers from 0.704 to 10 000.

The data from this experimental work was compared with previous studies for both free convection within an enclosure and free convection into an infinite atmosphere. Empirical equations were derived from this data, but, more important, was the development of criteria for determining whether a system should be considered enclosed or infinite.

For an infinite atmosphere, correlation of data taken from a variety of shapes by Nusselt, McAdams and King [1] shows that, for $Ra > 10^4$, the following equation applies:

$$Nu_D = 0.52(Ra_D)^{0.25} \tag{1}$$

where the characteristic dimension, D, is the diameter for cylinders and spheres, and the side length for cubes.

In a study restricted to horizontal pipes in air and water, McAdams [2] recommended using a coefficient of 0.53 for Ra ranging from 10^3 to 10^9. He also gives the results of an analytical study by Hermann, which yielded a coefficient of 0.40.

Lienhard [3] did both theoretical and experimental work on external free convection using a variety of shapes. He equated the drag force of the body on the boundary layer with the buoyant force of the boundary layer on the body and recommends using equation (1) with the boundary layer length, b, as the characteristic dimension (b is the distance traveled by the boundary layer around the inner body). He experimented with vertical plates, horizontal cylinders and spheres and found that the equation accurately predicted the heat transfer.

Studies have also been done on natural convection within an enclosure, showing that the same dimensionless parameters that were used to correlate convection to an infinite atmosphere (the Nusselt, Grashof, Prandtl and Rayleigh numbers) also work well in correlating the results of experiments on convection within an enclosure. In addition, some dimensionless ratio of characteristic dimensions is needed. The general form of the equation is the same as equation (1), with the ratio of characteristic dimensions as an additional parameter.

Scanlan *et al.* [4] correlated free convection between two concentric spheres. Warrington and Powe [5] extended the data to include convection from cubes, cylinders and spheres to a cubical enclosure. By defining the hypothetical radius as the radius of a sphere of the same volume as the body in question, they were able to correlate all available enclosure heat transfer data using the hypothetical gap width ratio, $(R_o - R_i)/R_i$ or L/R_i, as the geometric parameter (R_i and R_o are the hypothetical radii of the inner and outer bodies, respectively). They correlated the available data with

$$Nu_b = 0.590(Ra_b)^{0.235}(L/R_i)^{0.201} \tag{2}$$

with an average percent deviation of 14.54. Because the exponents on the Rayleigh number and hypothetical gap width ratio are nearly the same, the two parameters can be combined with only a small loss in accuracy. This simplifies equation (2) to

$$Nu_b = 0.585(Ra_b^*)^{0.236} \tag{3}$$

where Ra^* is the modified Rayleigh number, $(Ra)(L/R_i)$. This equation had an average percent deviation of 14.75.

On examination of Warrington and Powe's equations (2) and (3), it can be observed that for large values of L/R_i the Nusselt number becomes unbounded. Powe [6] made this same observation of Scanlan *et al.*'s [4] equations and noted that the Nusselt number should be bounded by the Nusselt number yielded by the equations for convection into an infinite atmosphere. Using equations for enclosures and for an infinite atmosphere, Powe noted that the enclosure equation could be used until the gap width ratio was between 1.3 and 2.2 for air and 2.0 and 4.0 for water. The exact value of L/R_i at which he recommended switching the infinite atmosphere equation was a function of the Rayleigh number.

Powe also noted that for small values of L/R_i heat transfer would be primarily due to conduction rather than convection. He recommended that the enclosure equation be used until the gap width ratio was reduced to where the equation for pure conduction predicted a higher heat transfer, at which point the conduction equation should be implemented. Warrington *et al.* [7] presented solutions for the conduction from bodies of various shapes to their enclosure. By evaluating conduction solutions and comparing them to the equation for convection into an enclosed fluid, it can be determined whether convection or conduction dominate as the means of heat transfer. It is recommended that one use whichever predicts the higher value for heat transfer.

Very little data are available for free convection within enclosures for large values of L/R_i, where the equations for enclosures and those for an infinite atmosphere predict the same value for the Nusselt number. The purpose of this study is to investigate that area. A detailed description of the apparatus comprising the test space, the experimental procedure, data reduction, and experimental uncertainty is given by Warrington and Powe [5] and will not be repeated here.

HEAT TRANSFER AND TEMPERATURE PROFILE RESULTS

Most of the existing equations for free convection into an infinite atmosphere yield similar results for the range of Rayleigh numbers of concern in this study, so one, equation (1), was chosen as a representative standard for free convection into an infinite atmosphere. This equation, reported by Jakob [1], represented the correlation of data from several different shapes, including those used in this investigation. Warrington and Powe [5] developed his empirical equation from a wide range of data from several sources, therefore, equation (3) was used as the standard for natural convection within enclosures.

To compare the two equations, equation (1) was rewritten using the distance travelled by the boundary layer as the characteristic dimension. For convection from cubes, the infinite atmosphere equation becomes

$$Nu_b = 0.618(Ra_b)^{0.25}. \tag{4}$$

As L/R_i increases, the heat transfer predicted by the enclosure equation increases without bound. Since the infinite atmosphere equation predicts the heat transfer for infinite L/R_i, it should form the upper bound for the enclosure equation. Equating Nusselt numbers from equations (3) and (4) then yields the lower limit of applicability of equation (4) as

$$(L/R_i) = 1.26(Ra_b)^{0.0593}. \tag{5}$$

The values of L/R_i defined by this transition range from 2.5 to 5.0, which is higher than the range of 1.3–4.0 as found by Powe [6]. It should be noted that Powe based his calculations on data from spheres only.

Based on the results of the above analysis, seven inner bodies were tested to provide data for L/R_i ranging from 2.28 to 5.89 and Rayleigh number ranges of 4.1×10^5–1.4×10^{10} (based on b) and 2.9×10^4–9×10^8 (based on D). The data taken from these seven bodies were correlated in several ways [5] using standard least-squares curve fitting techniques. The exclusion of the geometric factor (L/R_i) had little effect on the accuracy of any of the equations. This indicates that, for this range of L/R_i, the enclosure has little effect on the heat transfer. Warrington and Powe [5] found that for small L/R_i the geometric parameter had a much larger effect. Excluding

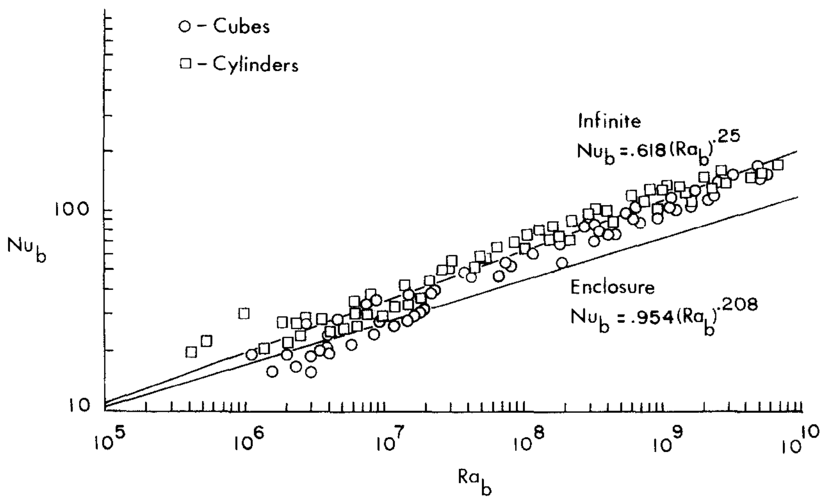

FIG. 1. Correlation of all transition region data with enclosure and infinite atmosphere equations.

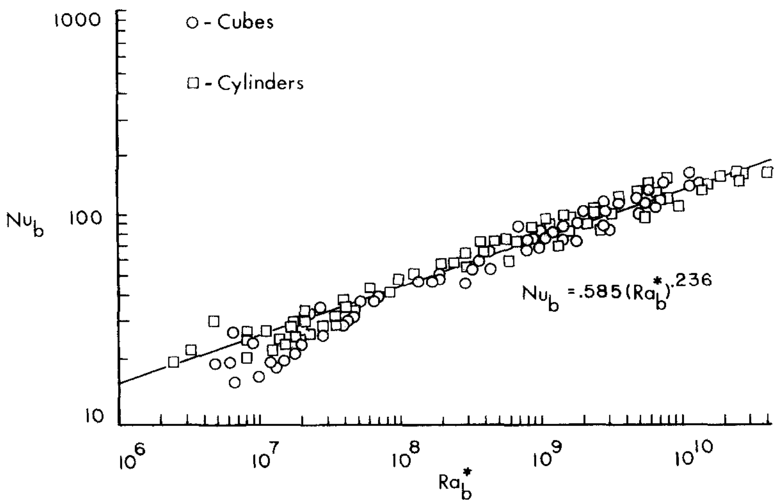

FIG. 2. Correlation of all transition region data with enclosure equation.

the geometric factor, the best equation for an enclosure was

$$Nu_b = 0.954(Ra_b)^{0.208}. \quad (6)$$

This equation had an average percent deviation of 18.51.

Figure 1 shows all of the transition region data compared with the enclosure equation (6) and the infinite atmosphere equation (4). It should be noted that this enclosure predicts a lower Nusselt number than either the infinite atmosphere equation or the data indicate. This is as expected since equation (6) was obtained from data where the gap width ratio was small, and the equation form does not account for changes in the gap width ratio.

The best enclosure equation (3) includes the geometric parameter, and Fig. 2 shows that the correlation was greatly improved by adding this parameter. Table 1 shows that this form of the equation has nearly the same accuracy as the infinite atmosphere equation.

Table 1. Correlation comparison between infinite atmosphere equation and enclosure equation

	Infinite atmosphere equation $Nu_D = 0.52Ra_D^{0.25}$		Enclosure equation $Nu_b = 0.585(Ra_b^*)^{0.236}$	
Data included	Average percentage deviation	Percentage of data within $\pm 20\%$ of equation	Average percentage deviation	Percentage of data within $\pm 20\%$ of equation
All data	13.23	80.84	12.71	84.94
Cubes	14.65	74.32	15.06	78.38
Cylinders	11.20	86.96	10.83	90.22
All data:				
$L/R_i < 1.26Ra_b^{0.059}$			11.96	90.11
$L/R_i > 1.26Ra_b^{0.059}$	12.17	84.00		

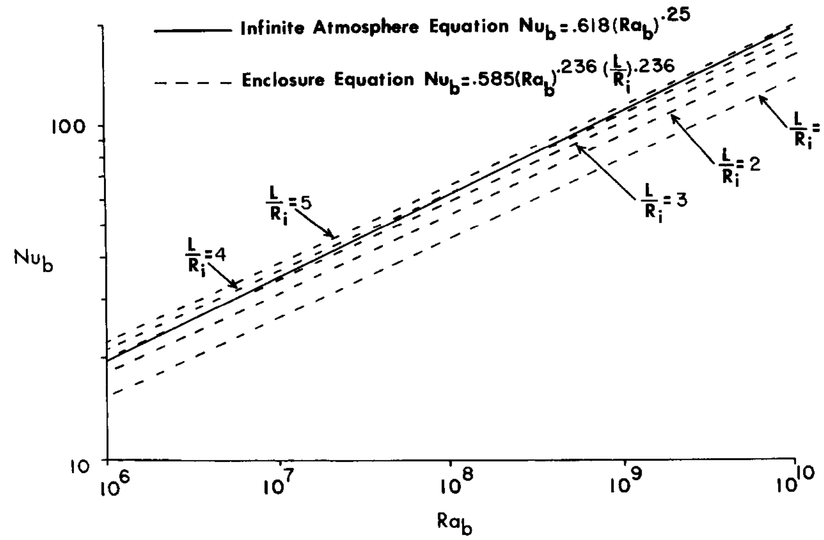

FIG. 3. Comparison between infinite atmosphere and enclosure equations.

Figure 3 shows that the best enclosure equation (3) accounts for the effect of changing L/R_i. As L/R_i increases into the transition region, both the enclosure equation (3) and the infinite atmosphere equation (4) predict nearly the same heat transfer. Either equation could be used in this region, but Table 1 shows that the use of equation (5) as the transition criterion improves the correlations. By including only those points consistent with the transition criterion, the average percent deviation involved in using the infinite atmosphere equation is decreased from 13.23 to 12.17%, and the enclosure equation deviation is reduced from 12.71 to 11.96%.

Temperature distributions within the gap were obtained at two vertical planes and at five angular positions in each plane. Figure 4 shows the temperature profile for the 6.67 cm cube in air. In this figure the temperature ratio is the local temperature minus the outer body temperature divided by the temperature difference between the inner and outer bodies and the radius ratio is the local radius minus the distance from the center of the inner body to the surface of the inner body divided by the difference in radii of the outer and inner bodies (all measured at the same angular position) [5]. According to equation (5), the system shown in Fig. 4 is very close to the transition between the enclosure and infinite atmosphere regions. This profile exhibits characteristics of the infinite atmosphere profiles as presented by Holman [8] and the enclosure profiles presented by Warrington and Powe [5].

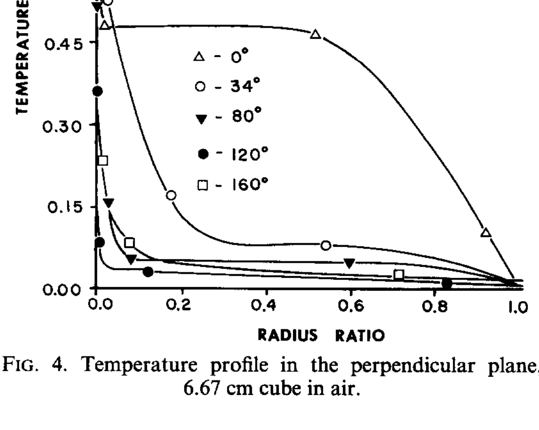

FIG. 4. Temperature profile in the perpendicular plane, 6.67 cm cube in air.

CONCLUSIONS AND RECOMMENDATIONS

This study has increased the amount of heat transfer and temperature profile data by extending the range of the hypothetical gap width ratio beyond that studied previously. This has increased the utility of the existing equations for natural convection both within an enclosure and in an infinite atmosphere by defining the range of hypothetical gap width ratios over which each are valid. The authors recommend Warrington and Powe's [5] equation (3) for convection within enclosures and Jakob's [1] equation (1) for convection to an infinite atmosphere. The authors recommend the use of equation (5) as the upper limit for use of the enclosure equation.

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